Some observations on the recent mortality trends in the Czech Republic, Slovakia, Poland and Germany.

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Abstract

This work examines the mortality characteristics of the populations of Czechia, Slovakia, Poland and Germany during the first years of the 21st century. The life tables of the male and female population of these countries were constructed and several mortality indicators such as the life expectancy at birth, modal and median age at death, Heligman-Pollard parameters etc. were studied. For the purpose of analysis, a statistical technique for smoothing the age-specific death rates in order to estimate the theoretical age-specific probabilities of dying was developed. This technique consists of a combination of the nine-parameter version of Heligman-Pollard formula (1980) proposed by Kostaki (1992) and three cubic splines. The results of our calculations are indicative of the mortality transition observed in these counties during the first years of the 21st century and the differences and similarities existing among their populations.

Key words:life tables, mortality indicators, Heligman-Pollard formula, cubic splines.

JEL Code:J1, J19, I1

Introduction

At the dawn of the 21st century in most of the countries of the world mortality transition is still evident with several improvements in mortality reduction expressed either by the life expectancy at birth and other ages or other indicators like the modal age at death. In this course the bimodal form of the life table death distributionoriginally described by Lexis (1878), in which the first mode corresponded to the infant and the second one to the old age mortality, has shifted towards more modern forms. This is due to the significant decrease in infant mortality and consequently the “concentration” of mortality at the older ages (Canudas-Romo 2008). Several hypotheses have been proposed in order for these developments to be explained,for instance the “mortality shifting hypothesis” (Bongaarts and Feeney 2002, 2003; Bongaarts 2005) and the “rectagularization of survival curves hypothesis” (see Wilmoth and Horiuchi 1999; Robine 2001). In the last hypothesis, the human survival curve becomes more rectangular as mortality levels decrease and this is closely connected to the notion of compression of mortality when a given proportion of deaths (in the old age heap) takes place at a shorter interval than before (Kannisto 2000).

In this context the question that will be addressed in this paper deals with the recent developments of mortality in the CzechRepublic and the surrounding countries: Germany, Poland and Slovakia. However, it is of equal importance to present the technique used for smoothing the age-specific death probabilities in order to estimate the theoretical ones. This technique consists of a combination of the nine-parameter version of Heligman-Pollard formula (1980) proposed by Kostaki (1992) and three cubic splines and will be presented in the methods and data session of this paper.

The reason for developing a new approach is that the methods used in the literature either fail to interpret the recent old age mortality trends [like the Gompertz (1825) method as tested in several countries like Greece, Germany etc.] or they are very complicated in their application (some other examples are the methods of Gompertz-Makeham 1860; Weibull 1951; Peristera and Kostaki 2005, Ouellette and Bourbeau 2011 etc.). On the contrary, the modified Heligman-Pollard formula has the ability to study parametrically several aspects of human mortality, like the accident hump between the ages 10 and 40, which is usually not taken into consideration in the vast majority of publications. However, as will be seen later, the third of the components of which this formula consists, is a typical Gompertz one and because of that several significant deviations can occur between the observed and smoothed data atthe older ages. Thus, three cubic splines were used after a certain age as they do not require any prerequites in their application while they are flexible enough to adapt to any mortality pattern and to be used in several intra and extrapolation techniques.

1 Data and Methods

Data come from the Human Mortality Database (www.mortality.org) and cover the first 15 years of the 21st century (2000-2014) except for Germany for which the available data is until the year 2013.

The procedure for smoothing the observed death probabilities, using the published full life tables, first of all is based on the Heligman-Pollard formula (1980) as modified by Kostaki (1992):

$$\frac{q_x}{p_x} = \begin{cases} A^{(x+B)^C} + De^{-E_1(lnx - lnF)^2} + GH^x, & for\ x \leq F \\ A^{(x+B)^C} + De^{-E_3(lnx - lnF)^2} + GH^x, & for\ x > F \end{cases}$$

As seen in the equation above the odds of mortality at age x are described by the summation of three components. The first component from the left to the right of the formula includes the parameters *A, B, C* and deals with mortality during childhood. However, because of the space limits of the paper this component will not be discussed. The third component includes the parameters *G* and *H,* corresponds to a Gompertz exponential and represents the aging or the deterioration of the human body. When the Heligman-Pollard modified formula was applied to the whole age range of the age specific death probabilities in the populations studied significant deviations were observed between the observed and the smoothed data. Consequently, it cannot be discussed and in reality this was the reason for the use of the cubic splines in the smoothing process that will be discussed later.

The second component deals with the accident hump at the ages 10-40 years. This hump appears either as a distinct hump in the mortality curve or at least as a flattening out of the mortality rates. It includes the parameters *D*, E_1, $E_2$and *F*.The parameter *D* represents the severity of the accident hump, and it will be discussed in this paper. If *F* denotes the location of the hump, E_1denotes its spread to the left and $E_2$to the right.

It was found during the analysis that the Heligman-Pollard modified method produced excellent fits until the age of 40-45 and sometimes 50 years, but significant deviations occurred afterwards. In order to solve the problem three subsequent cubic splines[1], i.e. three order polynomials,were used in the form:

$$\hat{q}_i = \hat{q}_x + a_k(x_i - x) + b_k(x_i - x)^2 + c_k(x_i - x)^3$$

where k=1…3 the number of spline, x_i the age and $\widehat{q_x}$ is the fitted value for $x=x_i$. Obviously the end of each spline is the beginning of the next one, while the knots were dynamically chosen in order for the best fit of the process to be achieved.

This process was carried out with the EXCEL SOLVER in order for the Sum of Squared Errors of the fit to be minimized. For the modified Heligman-Pollard formula the following term was chosen to be minimized:

$$\sum_x \left(\frac{\widehat{q_x}}{q_x} - 1\right)^2$$

where $\widehat{q_x}$ is the fitted value for age x and q_x is the observed one. A crude estimation of the fit was given by the R^2 estimator which tended to be almost 1 in both genders for all of the years and populations studied. A glimpse of this process is seen in Figures 1 and 2.

[1]see also http://mathworld.wolfram.com/CubicSpline.html

Fig. 1: The observed and smoothed probabilitied of death. Germany, males, 2008.

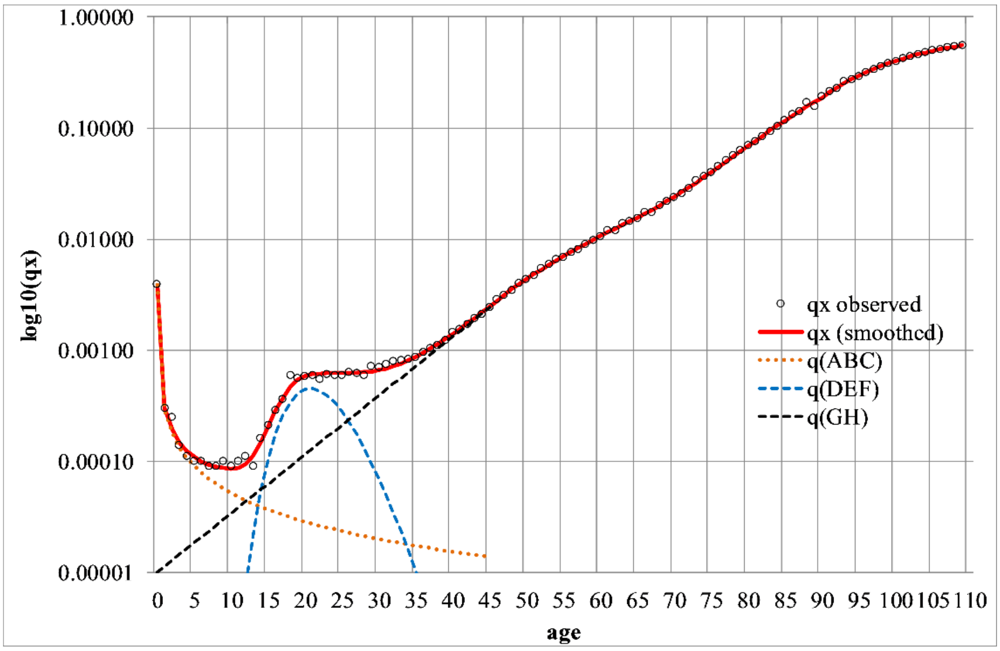
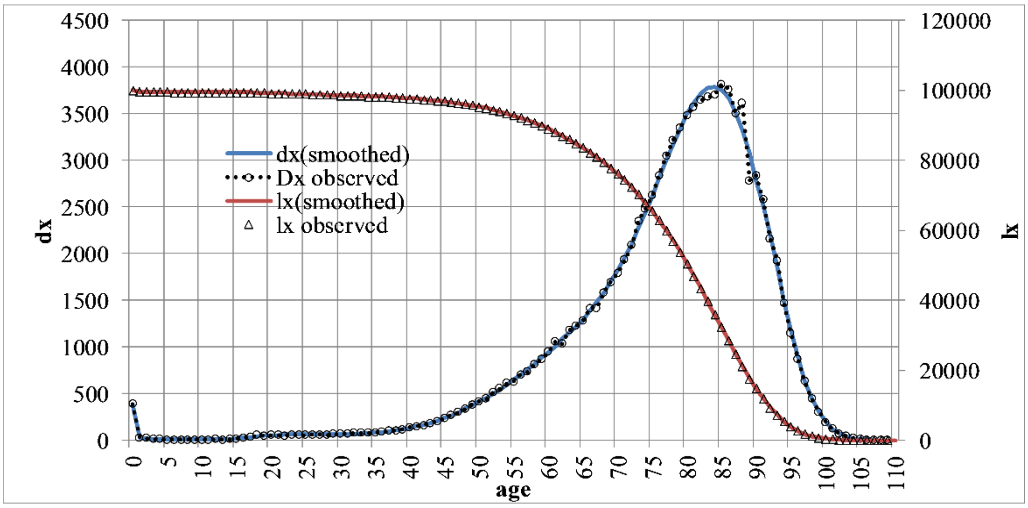


Fig. 2: The observed and smoothed lx and dx distributions. Germany, males, 2008.



In this paper the life expectancy at birth,which is a measure of the average length of life,will be studied. The *median age at death* was also studied, after the recalculation of the life table columns following the smoothing process, as happened with the other variables studied. It represents the age at which half of the deaths occur in the human life span and it was calculated according to the following formula (Canudas-Romo 2010):

$$Md(t) = x + \frac{[0.5 - l(x,t)]}{[l(x+1,t) - l(x,t)]}$$

where t is the year, x and $x+1$ are the ages of the interval at which the number of survivors equals 50% and l represents the number of survivors.

However, life expectancy at birth and the median age at death are affected by the infant, child and reproductive-adult mortality.Because of that the *modal age at death*, denoted as $M(t)$, is considered to be a better indicator of mortality shifts and longevity in the modern era, as determined only by old-age mortality(see Horiuchi et al. 2013). This quantity was calculated as (Kannisto 2001, Canudas-Romo 2008, 2010):

$$M(t) = x + \frac{[d(x,t) - d(x-1,t)]}{[d(x,t) - d(x-1,t)] + [d(x,t) - d(x+1,t)]}$$

where t is the year, x is the age where the maximum number of deaths occurs (except for the infant mortality) and d represents the number of deaths. Also the maximum number of deaths in the old age heap will be studied.

Finally, the Kannisto C family indicators will be studied as a measure of mortality compression according to the methodology which is described in the relevant paper (Kannisto 2000). Of them only the C50 indicator will be discussed, representing the narrowest age interval in which 50% of the deaths occur.

2 Results and Discussion

Life Expectancy at Birth in the 4 countries studied increases almost linearly in the first 14-15 years of the 21st century in both genders (Figure 3). However, Germany, where mortality transition has moved well ahead, is clearly differentiated. In females, Czechia and Poland form a group which fills the gap with Slovakia, where the highest mortality is observed. In males, after Germany, Czechia has lower mortality than the group of Poland and Slovakia.

During this course of mortality transition, females have lower gains in all countries than males. In Germany, between 2000 and 2013, these gains are 1.9 years in females and 3-3.5 years in the rest of the countries (2000-2014). In males, where mortality is always higher than females, these figures range between 4.2 years in Germany and 6.2 in Slovakia.

Fig. 3: Life expectancy at birth. Czechia, Slovakia, Poland and Germany. 2000-2014.

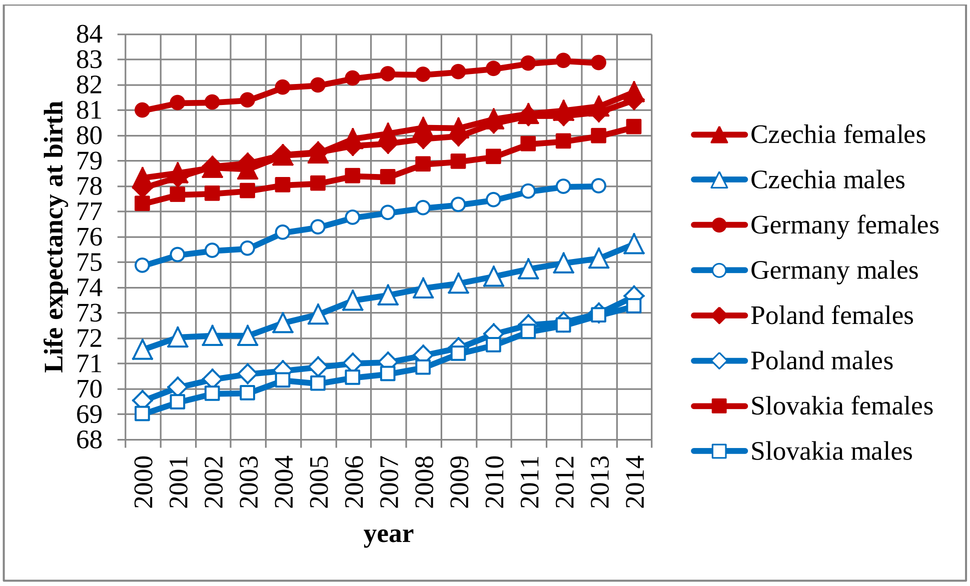
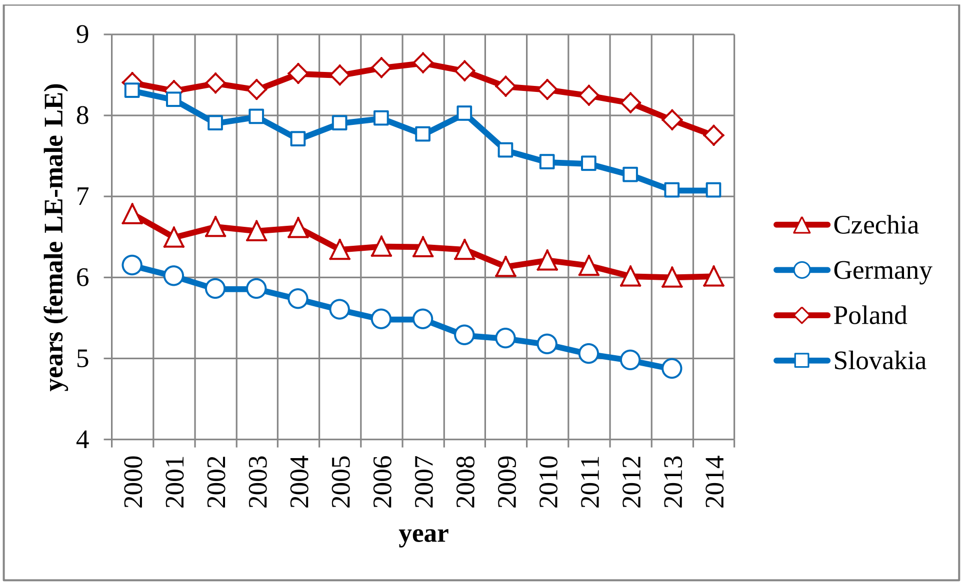


Fig. 4: The differences of life expectancy at birth among the two genders (females-males). Czechia, Slovakia, Poland and Germany. 2000-2014.



This transition is accompanied by the reduction of the differences existing among the two genders, even if this trend is not necessarily linear in all the counties studied (Figure 4). As a result, male excess mortality still persists, but the differences become as low as 4.9 years in Germany (2013) and as high as 7.8 years in Poland (2014).

A similar classification of the countries is observed if the median age at death is taken into consideration, and thus the relevant figures are not cited here; however, this is not the case with the modal age at death where greater differences are observed (Figure 5).

Fig. 5: Modal age at death. Czechia, Slovakia, Poland and Germany. 2000-2014.

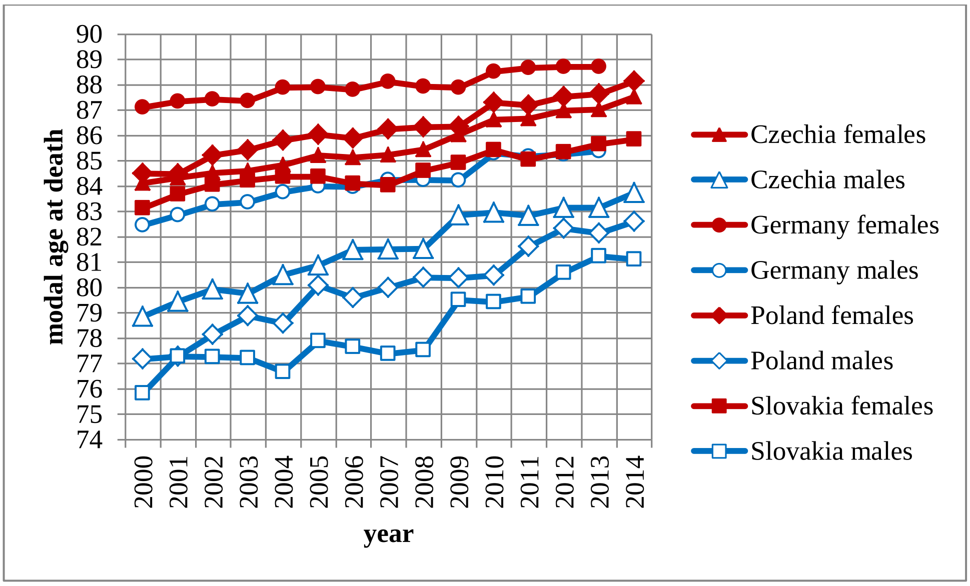
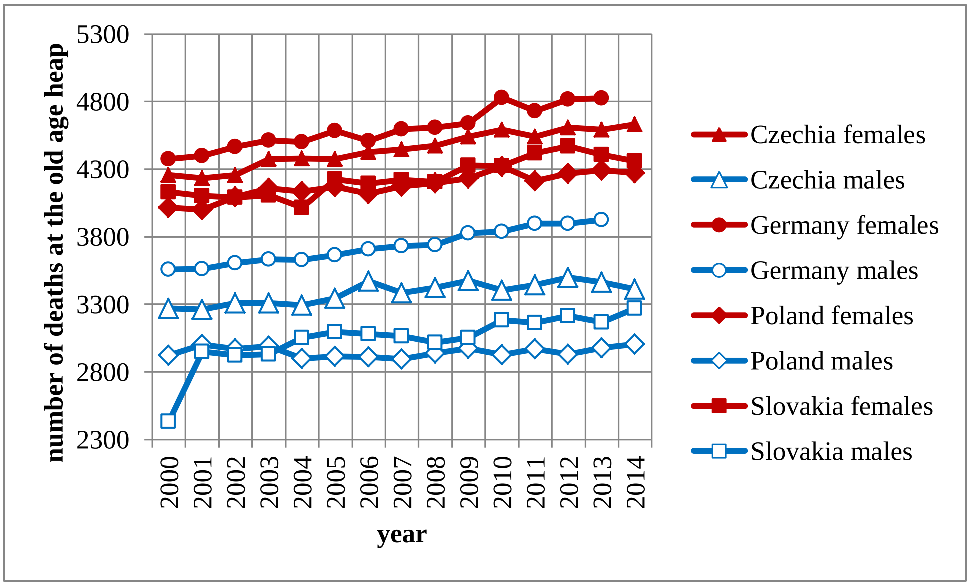


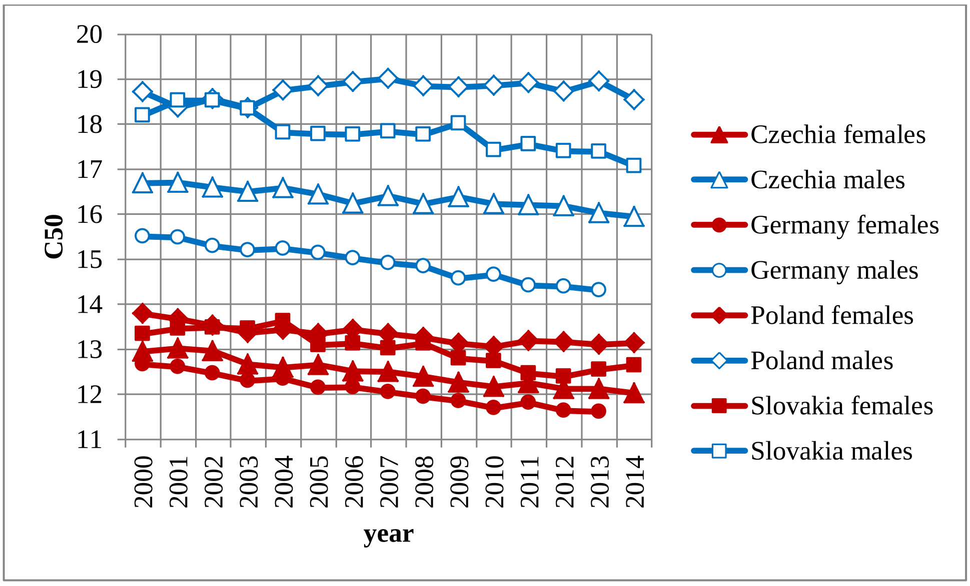
Fig. 6: Maximum number of deaths at the old age heap. Czechia, Slovakia, Poland and Germany. 2000-2014.



These differences are related to the form of the life table's death distribution in the old age heap and the developments of longevity, as measured with this variable, are not linear. Instead, several fluctuations occur through time; though the metathesis of the peak of the death curve in the older ages is more than evident. It seems that as mortality decreases, a drift of the modal age at death is caused. It has also to be noted that after 2005 the males' modal age at death of Germany largely follows the temporal trends and in some years practically coincides with that of females living in Slovakia, indicating on the one hand their differences in mortality and on the other hand the existing variability among the countries studied.

This variability is also connected with the maximum number of deaths at the old age heap, which is at increasing order besides the observed fluctuations in all the populations(Figure 6). However, in the last years of the study a small reversal trend is observed in the females from Slovakia and the males from the Czech Republic. In other words, as the mode of deaths is moving towards older ages over time the maximum values of the death distribution become higher. Quite expectedly the peak of the old age death heap is higher in the lower mortality populations, as it is in females in comparison with males.

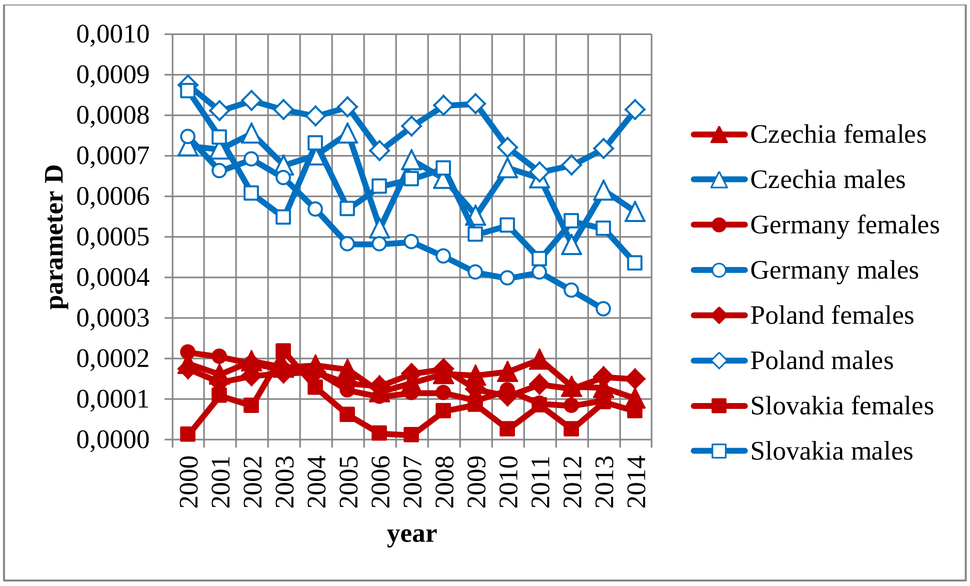
Fig. 7: The C50 Indicator. Czechia, Slovakia, Poland and Germany. 2000-2014.



Additionally, a trend for the compression of mortality is observed in the populations studied, as revealed by the Kannisto's family indicators C10, C25 and C50. As already mentioned, in this paper, only the C50 indicator will be cited due to the size limitations (Figure 7). The population of Poland (males and females) may constitutean exception as for several years the C50 indicator does not change significantly. Also, in the females from Slovakia a small increasing trend is observed for the last years of the study. In conclusion, despite the fact that the repertoire of mortality transition may exhibitsignificant peculiarities, the general trend is that the old age heap in most of the populations tends to become narrower over time, thus the death curve at the old ages tends to become steeper and the survival curve more rectangular.

The compression of mortality is more evident in the female population, as mortality is lower in them; however, the range in the male population is 15.5-18.7 years in 2000 and decreases between 14.31 (Germany 2013) and 18.54 (Poland 2014). In females the relevant figures are 12.67-13.79 years and 11.61 (Germany 2013) and 13.15 (Poland 2014) years respectively.

Fig. 8: The parameter D of the modified Heligman-Polard model. Czechia, Slovakia, Poland and Germany. 2000-2014.



Besides the old age mortality discussed above, an open question which needs to be discussed deals with the accident hump which occurs at the ages 10 to 40 and is described by the second component of the Heligman-Pollard model. As seen in Figure 8, the parameter *D*, which denotes the severity of the accident hump, is clearly lower in females than in males in all the populations studied; thus males' overall mortality levels are incurred a greater degree by the mortality in the younger ages of the human life cycle. This must be attributed to the behavioral, life style, occupational and other differences existing among the two genders.

However, there is a clear tendency for the reduction of the severity of the hump over time, despite some fluctuations which are observed in some male populations; it seems that males overtime tend to live a safer life in all the populations studied, apart from Poland's, where for the last years of the study the parameter *D* increases. Germany, also, is clearly distinguishable from the other countries. In females, several fluctuations of *D* are observed over time, but the severity of the accident hump is at all times small. The accident hump location denoted by the parameter F (not cited due to space limitations here), is between 20-23 years for males and 20 or below years for females.

Conclusion

A new method for the smoothing of death probabilities was applied and gave excellent results. This method has the advantage of combining the benefits of the Heligman-Pollard modified formula with the flexibility of the cubic splines.

The mortality transition in the countries studied is going on. It is faster in males than in females as it is in the higher than in the lower mortality countries. Life expectancy at birth and median age at death increase in all the populations. The same happens to the modal age at death, though significant fluctuations are observed under the general umbrella of the drift of that age towards older ones. At the same time the maximum number of lifetable deaths increases over time, even if some variability is observed. In that way the mortality curve at the old age heap tends to become higher over time, and also becomes narrower as the C50 indicator reveals, in most of the years and populations. Thus a clear trend of rectagularization is observed in most of the survival curves. Finally, the accident hump becomes over time less important in all the populations studied, besides the observed variability. The effects of this hump, are more important in males than in females.

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